Noise can delay and advance the collapse of spatiotemporal chaos

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Spatiotemporal chaos on a regular ring network of excitable Gray-Scott dynamical elements collapses to a stable asymptotic state. We find that the addition of dynamical noise clearly influences the spatiotemporal pattern and the transient lifetime of spatiotemporal chaos. Spatially uniform noise significantly decreases the average lifetime of spatiotemporal chaos due to an enlargement of regions of local collapse. For spatially inhomogeneous noise the collapse is maximally delayed at an intermediate noise level, but drastically advanced for larger noise levels.

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I. INTRODUCTION

Transient spatiotemporal chaos is a common phenomenon in extended nonequilibrium systems across several disciplines. In the absence of external perturbations, complex spatiotemporal dynamics changes spontaneously from chaotic to steady state or periodic behavior. In contrast to low-dimensional systems where a chaotic repellor is known to govern transient chaos [1], the mechanistic understanding of the collapse in extended systems remains elusive. Transient spatiotemporal chaos has been reported in models for turbulent dynamics [2], for semiconductor charge transport [3], for CO oxidation on single-crystal Pt surfaces [4], for a cubic autocatalytic mass-action model [5], and in systems of coupled logistic maps [6,7]. In addition, spatiotemporal complexity with irregular dynamics and fast decaying correlations but a negative maximum Lyapunov exponent ("stable chaos") was found to be transient in systems of coupled one-dimensional maps [8,9]. Transient spatiotemporal chaos was also discussed as a mechanism for species extinction in ecology [10,11], motivated from the hypothesis that transient dynamics in ecological models might be more relevant for real systems than long-term model behavior [12].

The asymptotic stability of chaotic dynamics in extended systems is difficult to determine, since transient spatiotemporal chaos may be extremely long lived; its average lifetime typically increases exponentially with the size of the medium [3–5,9,13]. Noise and nonlocal coupling may further influence the collapse process in realistic systems, which enhances the difficulties in determining the asymptotic stability. For transient chaos in the Gray-Scott reaction-diffusion network it was demonstrated that (1) the addition of two or more nonlocal couplings in the network can make transient spatiotemporal chaos asymptotic [14], and that (2) introducing resource competition into the model causes species-segregation that prevents the collapse of spatiotemporal chaos [15].

The study of noise effects is fundamental to the understanding of realistic complex spatiotemporal behavior. The variety of noise-induced dynamical phenomena in extended systems is huge [16,17], including noise-supported traveling waves [18], noise-sustained coherent oscillations [19], noise-mediated synchronization [20], and noise-induced attractor switching [21]. In systems with spatiotemporal chaos, noise causes merging of turbulent waves [22], and noise enhances the creation and annihilation of topological defects [23].

Very few studies have addressed the collapse of spatiotemporal chaos in the presence of noise so far. From the robustness of the transient times in a noisy diffusively coupled logistic map lattice, Lai concludes for his model that the presence of noise is not advantageous in attempts to reduce the transient lifetime [24]. Even in low-dimensional systems, where the collapse mechanism for transient chaos is mathematically understood [1], the effects of noise are less clear, and the findings range from a reduction of the lifetime [25], to robustness against noise [26], to prolonged lifetimes in a post-crisis parameter regime [27].

In this paper we report that additive noise clearly affects the collapse of spatiotemporal chaos in a regular ring network of Gray-Scott excitable elements. The model is introduced in Sec. II. In Sec. III we demonstrate that spatially homogeneous dichotomous Markov noise advances the collapse and we discuss the responsible patterns in the noise realizations. In Sec. IV we show that spatially inhomogeneous noise can delay and advance the collapse process, depending on the amplitude of the noise.

II. THE MODEL

The (network) model consists of N diffusively coupled, identical, continuous-time dynamical elements. The excitable dynamics at each network node n ($n=1,2,\ldots,N$), is given by the two-variable Gray-Scott model [28], which describes an open, autocatalytic reaction: $A+2B\rightarrow 3B$ and $B\rightarrow C$. A represents the reactant (resource), B the autocatalytic species, and C the final product. This reaction at every node n is modeled by the following coupled dimensionless differential equations:

$$\frac{da_n}{dt} = 1 - a_n - \mu a_n b_n^2 + D\Delta_n(a_n),$$

$$\frac{db_n}{dt} = \mu a_n b_n^2 - \Phi b_n + D\Delta_n(b_n) + Q\xi_{n,t} f(n,t). \quad (1)$$

a_n and b_n are the dimensionless concentrations of resource A and species B at node n. Φ and μ are the bifurcation param-

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eters, determined by the rate constants and the reactant concentration in the reservoir. Diffusive coupling $\Delta_n(\ )$ with coupling strength D is assumed for both reaction variables, using $\Delta_n(x_n)=x_{n+1}+x_{n-1}-2x_n$.

Species concentration b_n at node n is perturbed with additive dynamical noise with noise amplitude Q and zero mean. For dichotomous Markov noise, the random variable $\xi_{n,t}$ is uniformly distributed in the set $\xi_{n,t}\in\{-1,1\}$, such that the noise distribution for $Q\xi_{n,t}$ is given by $h(Q\xi_{n,t}) =\frac{1}{2}[\delta(\xi_{n,t}-Q)+\delta(\xi_{n,t}+Q)]$. The function $f(n,t)=\tau(t)X(n)$ determines the time t and the location n for adding noise in the network [Eq. (1)]. Noise is added every t_0 time units, $\tau(t)=\delta_{t,i\times t_0}$ and $i=1,2,\ldots$. For *spatially homogeneous noise* the same random perturbation is added at every network node, i.e., $\xi_{n,t}X(n)=\xi_{1,t}$. For *spatially inhomogeneous noise* every network node is perturbed, but the network is divided into r regimes such that N/r neighboring nodes are perturbed with the same noise event. Then, $\xi_{n,t}X(n) =\xi_{j,t}\delta_{1,\Theta\{n-[(j-1)N]/r\}\Theta[(jN)/r-n]}$, with $\Theta(\ )$ the Heaviside function and $j=1,2,\ldots,r$. The degree of spatial inhomogeneity of the noise is controlled by r, with $r=1$ corresponding to spatially homogeneous noise, and $r=2$ corresponding to one noise realization in every half of the network.

In the absence of coupling ($D=0$) and in the absence of noise ($Q=0$) in Eq. (1), the dynamics at a network node is characterized by three steady states,

$$S^n=(1,0),$$

$$S^f=\left(\frac{1-\sqrt{1-4\Phi^2/\mu}}{2},\frac{1+\sqrt{1-4\Phi^2/\mu}}{2\Phi}\right),$$

$$S^s=\left(\frac{1+\sqrt{1-4\Phi^2/\mu}}{2},\frac{1-\sqrt{1-4\Phi^2/\mu}}{2\Phi}\right).$$

Linear stability analysis shows that S^n is a stable node for all parameter values μ and Φ. S^f is an unstable focus, and S^s is a saddle point, which exist for μ above the saddle node bifurcation point, $\mu_{sn}=4\Phi^2$. In the range $2<\Phi<4$, S^f becomes a stable focus above the subcritical Hopf bifurcation point, $\mu_H=\Phi^4/(\Phi-1)$. In the parameter regime of interest $[\mu_{sn},\mu_H]$ and $\Phi=2.8$ the dynamical system at each node is excitable.

Earlier studies have shown that the network dynamics on a regular ring network [Eq. (1)] exhibits transient spatiotemporal chaos in the absence of noise ($Q=0$) [5]. After a regime of sustained spatiotemporal chaos with a rapid decay of spatial correlations and a positive largest Lyapunov exponent, the system exhibits a spontaneous, intrinsic collapse to the homogeneous stable steady state S^n with extinct species. The average lifetime of transient spatiotemporal chaos increases exponentially with the size of the network N [5]. During the transient phase the spatiotemporally chaotic dynamics was characterized by a Šilnikov-like orbit that consists of a heteroclinic connection from the unstable focus S^f to the stable node S^n in the homogeneous system and a heteroclinic connection from S^n to S^f for the traveling wave system [29,30]. A typical trajectory at a network node spirals

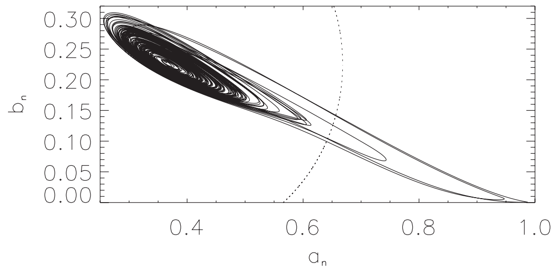

FIG. 1. Characteristic phase portrait for Eq. (1) with system parameters $\mu=33.7$, $\Phi=2.8$, $D=16$ [31], and $Q=0$. The dashed line marks a distance $d=0.3$ to the unstable focus.

away from the unstable focus toward the stable node, only to be reinjected to the unstable focus via the propagating reaction-diffusion activity (Fig. 1). The parameter range $[\mu_c,\mu_H]$ for wave-induced spatiotemporal chaos is determined by the critical threshold for traveling wave solutions μ_c and the Hopf bifurcation point μ_H, with $\mu_c\approx 33$ for $\Phi=2.8$ [29].

The spatiotemporal pattern during the transient phase is characterized by an irregular distribution of local extinct regions, in which trajectories of neighboring network nodes approach the stable steady state S^n of the homogeneous system together [29]. The triangular shape of these local extinctions [[5], similar to Fig. 8(a)], where species B is extinct ($b=0$) and resource A recovers to its maximum value ($a=1$), is due to the propagation of species B into these regions of high resource concentration A from both sides. The collapse of spatiotemporal chaos is preceded by a quasihomogeneous spatial state (basin for immediate extinction in [5]) in which the perturbations that normally initiate reaction-diffusion fronts and thus sustain spatiotemporal chaos are subthreshold. The trajectories throughout the network follow closely the heteroclinic connection from the unstable focus to the stable node, and the system reaches its stable, spatially homogeneous asymptotic state.

In all simulations in this paper, the noise amplitude Q was chosen clearly below the excitation threshold of the Gray-Scott dynamics to explore noise-induced changes of the collapse dynamics and to avoid noise-induced excitation of the entire network dynamics.

III. THE COLLAPSE OF SPATIOTEMPORAL CHAOS IN THE PRESENCE OF SPATIALLY HOMOGENEOUS NOISE

Spatially uniform dichotomous Markov noise significantly decreases the average lifetime of spatiotemporal chaos (Fig. 2) for all noise amplitudes Q. The upper limit for Q, $Q\leqslant 0.01$ was chosen to ensure that noise provides only subcritical perturbations to the stable rest state S^n, i.e., a single noise event cannot reexcite the system by itself. Above a certain noise threshold, $Q>10^{-4}$, the collapse of spatiotemporal chaos is strongly advanced, and the average transient time $\langle T\rangle$ follows a power law decay for the larger network sizes, $N=140$ and $N=160$. For the small network size, $N=120$, a power law exists only approximately. For $Q<10^{-4}$

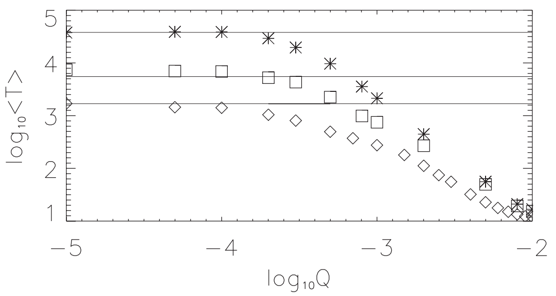

FIG. 2. Average transient lifetime $\langle T\rangle$ as a function of noise amplitude Q for different network sizes, $N=120$ (◇), $N=140$ (□), and $N=160$ (★). Spatially homogeneous dichotomous Markov noise with zero mean was added every $t_0=0.15$ time units to the species concentration b in Eq. (1). Control calculations show that $\langle T\rangle$ is robust when adding noise to the resource concentration a, instead. Each data point was determined from 10^3 simulations, using 100 different random initial conditions for spatiotemporal chaos and 10 noise realizations for each initial condition. The horizontal lines show the average transient time for 100 random initial conditions in the absence of noise. The error bar (1 standard deviation, not plotted here) for each data point is of the order of $\langle T\rangle$. For each simulation reactant A is initially distributed homogeneously ($a=1$) over the entire network, and species B is randomly seeded with an initial concentration of $b=1$. Equation (1) was integrated with an explicit Euler method, a numerical time step of $dt=0.0003$, and periodic boundary conditions; the system parameters are $\mu=33.7$, $\Phi=2.8$, and $D=16$ [31].

the average transient time $\langle T\rangle$ also decreases with noise amplitude Q, but it follows more an exponential decay. The change in the strength of the decay in the neighborhood of $Q=10^{-4}$ is independent of the network size, which points to its origin in the interaction of the noise with the local dynamics at each network node, and not to a global network phenomenon. The statistical robustness of the average transient lifetime was demonstrated in control simulations for various noise amplitudes Q [32]. In Fig. 2 noise was added every $t_0=0.15$ time units, which is small in comparison to the typical time scale of the system given by the excitation cycle. A variation of t_0 does not qualitatively change the noise-advanced collapse phenomenon in Fig. 2, but reveals that the average transient lifetime decreases with decreasing t_0 [33].

On a regular ring network of Gray-Scott excitable elements the lifetime of spatiotemporal chaos increases exponentially with the network size N [5]. Figure 3 shows for a representative noise amplitude ($Q=0.001$) that the exponential dependence persists in the presence of homogeneous dichotomous noise, but the increase is reduced in comparison to the noise-free case. Such an exponential increase of the average lifetime with medium size appears to be characteristic for various other noise-free diffusively coupled systems [3,4,9,13,14], its origin, however, remains elusive.

A statistical analysis of the transient lifetimes for different initial conditions and different noise realizations shows an exponential frequency distribution in the absence of noise and also in the presence of homogeneous dichotomous noise [Figs. 4(a) and 4(b)]. For a network of $N=120$ nodes the simulations yield a noise-free distribution that is characterized by a maximum transient lifetime of $T=11\,116$ and an

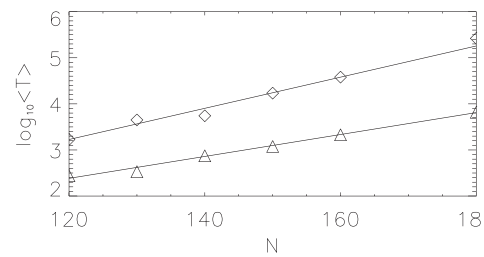

FIG. 3. Average transient lifetime $\langle T\rangle$ versus number of network nodes N in the absence of noise $Q=0$ (◇) and in the presence of spatially homogeneous dichotomous Markov noise with noise amplitude $Q=0.001$ (△). The full lines show "robust least absolute deviation" fits of the average transient lifetimes. All the other parameters and simulation procedures are the same as in Fig. 2.

average transient time of $\langle T\rangle=1688\pm1710$. The distribution in the presence of noise ($Q=0.001$) is characterized with a clearly reduced maximum transient time $T=2129$ and average transient time $\langle T\rangle=268\pm254$. The statistical analysis further reveals that not every noise realization is reducing the lifetime of spatiotemporal chaos, although the average transient lifetime is clearly reduced. Above a certain noise amplitude ($Q\geqslant0.001$ for $N=140$), however, our simulations show an advanced collapse of spatiotemporal chaos for all noise realizations.

Spatiotemporal chaos on a regular ring network [Eqs. (1)] is characterized by an irregular distribution of local extinct regions in space and time, in which neighboring trajectories collapse to the stable rest state S^n. First insight into the mechanistic role of noise for transient spatiotemporal chaos is gained from a statistical analysis of the sizes of local extinction. It reveals that noise causes more and larger regions of local extinction (Fig. 5). During a simulation time of 75 000 the spatiotemporal dynamics exhibits 5718 local extinctions with an average size of $\langle s\rangle=1105\pm1287$ in the

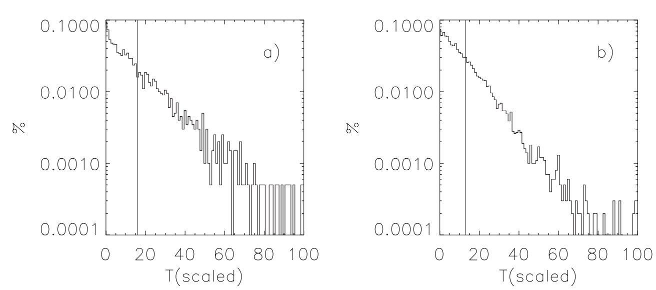

FIG. 4. Distribution of scaled transient lifetimes T for a network with $N=120$ nodes in the (a) absence of noise ($Q=0$) and in the (b) presence of homogeneous dichotomous noise with amplitude $Q=0.001$. The range of transient times was divided into 100 bins, starting with $T=0$ and ending with the maximum observed transient time, $T=11\,116$ in (a) and $T=2129$ in (b). The average transient time $\langle T\rangle$, marked with a vertical line, is $\langle T\rangle=1688\pm1710$ in (a) and $\langle T\rangle=268\pm254$ in (b). The histogram in (a) was generated from 2000 numerical simulations with different random initial conditions, and the histogram in (b) was generated from 10 000 simulations with 100 different random initial conditions and 100 noise realizations for each initial condition. All the other parameters are the same as in Fig. 2.

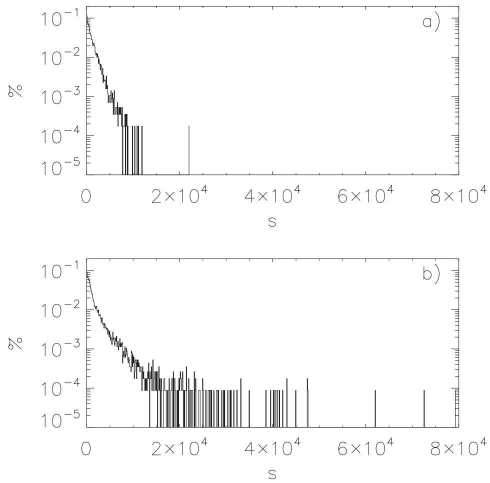

FIG. 5. Size distribution of local extinctions (triangular patches in the spatiotemporal pattern [5]) in the absence of noise (a) and in the presence of homogeneous noise [(b), $Q=0.001$]. A local extinction was defined by $a>0.8$, and an increased network size, $N=600$, was chosen to guarantee statistical significance of the distribution. The simulation time for both figures was 75 000 time units, resulting in 5718 clusters with an average size of $\langle s\rangle=1105\pm1287$ pixels in (a), and 11 437 clusters with $\langle s\rangle=1848\pm3488$ pixels in (b). For all the other parameters see Fig. 2.

noise-free case [Fig. 5(a)], 7859 local extinctions with an average size of $\langle s\rangle=1291\pm1871$ in the presence of dichotomous noise with amplitude $Q=0.0005$, and 11 437 local extinctions with an average size of $\langle s\rangle=1848\pm3488$ for a noise amplitude of $Q=0.001$ [Fig. 5(b)]. More and larger regions of local extinction have competing consequences for the lifetime of spatiotemporal chaos. The noise-induced trend for larger regions of local extinction [Fig. 5(b)] favors the global collapse of spatiotemporal chaos by increasing the chance for an extinction of the size of the network. More local extinctions, however, hinder the global collapse of spatiotemporal chaos, since the presence of a local extinction anywhere in the network prevents the collapse of spatiotemporal chaos in the entire network by triggering superthreshold perturbations (i.e., excitations) at the boundary of the local extinction [5]. These superthreshold perturbations also prevent the merging of local extinctions to a global extinction.

Insight into the origin of noise-mediated larger (and more frequent) regions of local extinction is gained from studying the distance of a typical trajectory to the unstable focus S^f (Fig. 1). It follows that the average time interval (residence time) between a trajectory's successive approaches to the stable node S^n is clearly reduced for increasing noise amplitude. For example, the average time of a typical trajectory in the neighborhood of S^f until it reaches a distance $d=0.3$ to the unstable focus (Fig. 1) is 160 ± 151 in the absence of noise, 98 ± 93 for $Q=0.001$, and 48 ± 46 for $Q=0.002$. This result in coupled Gray-Scott excitable elements is consistent with earlier findings in one-dimensional maps, where dynamical noise also decreases the average residence time of a trajectory in the neighborhood of an unstable steady state [34]. Since a typical trajectory under noise reaches a certain distance to the unstable focus ($d=0.3$ in Fig. 1) faster in

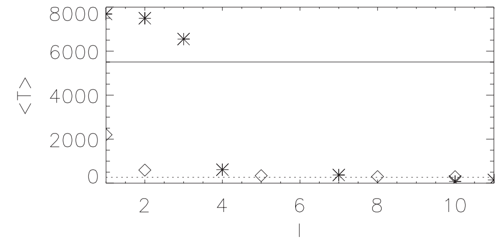

FIG. 6. Dependence of the average transient lifetime $\langle T\rangle$ on the length scale l of the dichotomous perturbation. For periodic dichotomous perturbations (★) the length scale is $l=p/2$ with p the period of the perturbation; for restricted dichotomous noise (□) the length scale l represents the maximum distance of a random walker from the origin after k steps, $l\geq\Sigma_{i=1}^{i=k}\xi_{1,i}$ with $k\in N$ and $\xi_{1,i}\in\{-1,1\}$. The amplitude of the perturbation is $Q=0.002$, and the size of the network is $N=140$. The full horizontal line shows the average transient time in the absence of perturbations, and the dotted line shows the average transient time in the presence of dichotomous Markov noise (from Fig. 2). All the other parameters are given in Fig. 2.

time, and since every trajectory in the network experiences the same noise realization, it is more likely that a larger number of neighboring trajectories reach a certain distance to S^f together, which is most advantageous for an immediate local or global extinction to occur. As shown in [5] for the noise-free system, spatiotemporal configurations with most of the trajectories a distance $d\approx0.1$ away from S^f experience an immediate collapse of spatiotemporal chaos, rather independent of the phase of individual trajectories relative to S^f. Since subsequences in the noise realizations with predominantly positive (or negative) events often move a trajectory away from S^f on either side, and since noise is spatially homogeneous over the network, such noise patterns act as a driver for all the trajectories to reach a sufficient distance to S^f to immediately approach the stable node S^n, following closely the heteroclinic connection from S^f to S^n in the homogeneous system [29].

To test for such favorable noise patterns that act as a driver for the spatiotemporal system to reach extinction, the sequences of dichotomous noise events are manipulated for a fixed noise amplitude, and the consequences for the lifetime of transient spatiotemporal chaos are determined. Figure 6 shows the average transient lifetime for homogeneous dichotomous noise, constrained by various length scales l. l represents the maximum distance of a random walker from the origin after k steps, $\Sigma_{i=1}^{i=k}\xi_{1,i}\leq l$ with $k\in N$ and $\xi_{1,i}\in\{-1,1\}$. The average transient time in Fig. 6 is rather robust for length scales $l>2$, and its value is close to the case of unrestricted noise. Reducing l from $l=2$ to $l=1$ clearly increases the average lifetime to an intermediate value between the noise-free case and the unrestricted noise case. $l=2$ ($l=1$) allows at most four (two) successive noise events with the same sign. This dependence of the collapse on the length scale l confirms that noise sequences that are dominated by either positive or negative noise events cause the advancement of the collapse process, because such noise patterns also move the trajectories a sufficient distance away from S^f on either side to initiate the collapse [33].

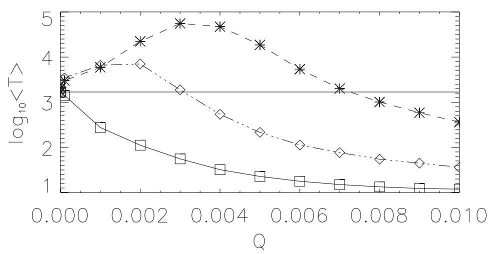

FIG. 7. Average transient time $\langle T\rangle$ versus noise amplitude Q for spatially inhomogeneous dichotomous noise. Noise is added at every location in the network ($N=120$). The degree of spatial noise inhomogeneity decreases from $r=20$ (★), to $r=4$ (◇), to spatially homogeneous noise for which $r=1$ (□). The horizontal line marks the average transient time in the absence of noise. For other dynamical, numerical, and statistical parameter values see Fig. 2.

Since subsequences with $l>0$ are present in other types of stochastic processes, we expect that uniform white noise, Gaussian white noise, or various colored noise realizations will also advance the collapse of spatiotemporal chaos. It further suggests that even periodic perturbations will advance the collapse process. Figure 6 shows the average transient time versus l for periodic dichotomous perturbations, where l is defined as $l=p/2$ with p the period of the perturbation. For $l\geqslant 4$ the average transient times for the periodic and the stochastic dichotomous perturbations do not differ. Periodic perturbations with $l\leqslant 2$, however, clearly delay the collapse of spatiotemporal chaos as seen from Fig. 6 for a network size of $N=140$ and from control calculations for $N=160$. A more detailed analysis shows that trajectories under periodic perturbations with small periods escape on average slower from the immediate neighborhood of the unstable focus S^f than trajectories with no perturbations. For example, the average residence time of a typical trajectory in the neighborhood of S^f ($d=0.3$, Fig. 1) is 160 ± 151 for $Q=0$, but 179 ± 152 for periodic perturbations with $l=1$ and $Q=0.002$. Thus periodic perturbations with small periods can hinder a trajectory to reach a certain distance to S^f and by this delay the collapse process.

IV. THE COLLAPSE OF SPATIOTEMPORAL CHAOS IN THE PRESENCE OF SPATIALLY INHOMOGENEOUS NOISE

Spatially inhomogeneous dichotomous Markov noise clearly influences the collapse of spatiotemporal chaos. Figure 7 shows that the average lifetime of spatiotemporal chaos first increases (up to several orders of magnitude) with increasing noise amplitude Q until it reaches a maximum value. Then the average lifetime decreases and drops below the lifetime for the noise-free dynamics. The delay of the collapse is more drastic and it happens over a larger range of noise amplitudes when the degree of spatial noise inhomogeneity is enlarged. For example, if 5% of neighboring network nodes are subject to the same noise realization ($r=20$) the maximum average lifetime is $\langle T\rangle=55\,550\pm55\,630$ (Fig. 7), whereas if 25% of neighboring network nodes are subject to the same noise realization ($r=4$) the maximum average lifetime is $\langle T\rangle=7047\pm7665$. In both cases the collapse is clearly delayed in comparison to the noise-free dynamics, where $\langle T\rangle=1688\pm1710$.

The presence of a maximum in the average transient time $\langle T\rangle$ in Fig. 7 is caused by two competing mechanisms. (1) $\langle T\rangle$ increases with noise amplitude Q: Larger noise amplitudes increase the spatial inhomogeneity of resource a_n and species concentration b_n throughout the network. This makes it harder for the system to reach the quasihomogeneous state, which was identified as an immediate precursor of the collapse [5]. In addition, additive spatially inhomogeneous noise directly enters into the Laplacian coupling term, $\Delta_n(\xi_{n,t})\neq 0$ for several n in Eq. (1), with the possibility to enlarge the coupling-induced perturbation that a trajectory at node n is experiencing. For spatially homogeneous noise, however, $\Delta_n(\xi_{n,t})=0$ for all n. A nonvanishing $\Delta_n(\xi_{n,t})$ increases the spatial inhomogeneity in the network, and can make the perturbation onto a trajectory super-threshold to sustain spatiotemporal chaos [35]. (2) $\langle T\rangle$ decreases with noise amplitude Q: Larger noise amplitudes reduce the average residence time of a trajectory in the neighborhood of the unstable focus. Noise patterns with predominantly positive (or negative) events often move a trajectory away from S^f on either side of the focus, which promotes the collapse process as discussed in Sec. III. The decay of $\langle T\rangle$ with Q in Fig. 7 exists for the entire amplitude range for spatially homogeneous noise ($r=1$), but is shifted towards larger noise amplitudes for spatially inhomogeneous noise ($r>1$) due to (1).

Figure 8 shows that spatially inhomogeneous dichotomous Markov noise alters the spatiotemporal pattern. With increasing noise amplitude more and larger regions of local extinction arise, and their triangular shapes become more distorted. The arguments that more local extinctions delay the collapse process, whereas larger regions of local extinctions advance the collapse process, as discussed in Sec. III, apply for spatially inhomogeneous noise as well and are consistent with the decay of $\langle T\rangle$ with Q in Fig. 7. Additional analysis shows that the frequency distribution of lifetimes for a given noise amplitude follows an exponential distribution in the presence of spatially inhomogeneous noise, as is the case for spatially homogeneous noise in Fig. 4.

V. CONCLUSIONS

Dynamical noise clearly influences the collapse of spatiotemporal chaos to a stable asymptotic state in a regular network of Gray-Scott excitable elements. Spatially uniform dichotomous Markov noise significantly decreases the average lifetime of spatiotemporal chaos with increasing noise amplitude. Spatially inhomogeneous dichotomous Markov noise can drastically delay the collapse of spatiotemporal chaos; the transient lifetime can increase up to several orders of magnitude with increasing noise amplitude. Above a critical noise amplitude, however, the collapse is also advanced for spatially inhomogeneous noise, and the average lifetime of spatiotemporal chaos drops below the lifetime of the noise-free dynamics. The advancement of the collapse for spatially homogeneous and spatially inhomogeneous noise is

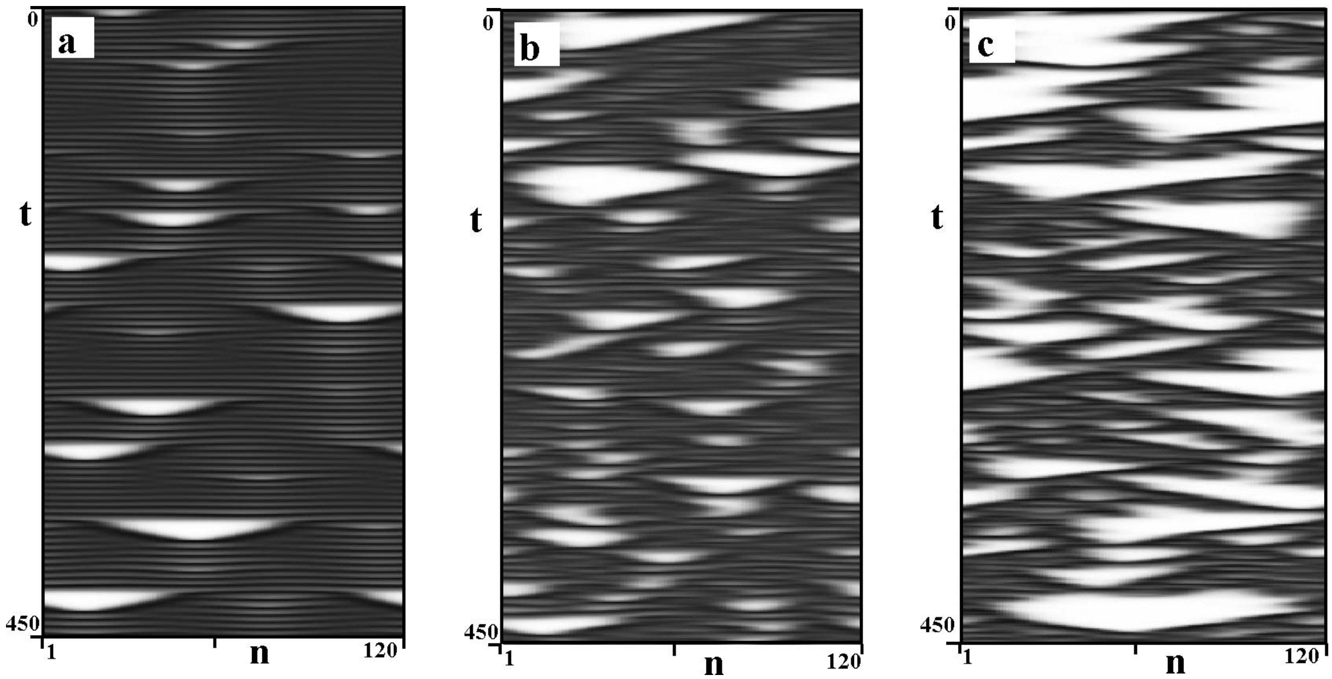

FIG. 8. Spatiotemporal pattern of reactant concentration a at network nodes n in the presence of spatially inhomogeneous dichotomous noise ($r=20$). Noise amplitudes are increasing from left to right: (a) $Q=0.0001$, (b) $Q=0.005$, and (c) $Q=0.009$. The network has $N=120$ nodes, and the simulation time is $\Delta t=450$. A concentration of $a=1$ ($a=0$) is represented in white (black). Other parameters and simulation procedures are the same as in Fig. 2.

due to certain noise patterns that enlarge regions of local collapse to increase the probability for a collapse of the size of the network.

The asymptotic stability of extended systems with diffusive coupling is difficult to determine, since the average lifetime typically increases exponentially with the size of the network or medium [3–5,9,13,14]. For the regular network of Gray-Scott excitable elements we find that this exponential increase still holds in the presence of noise, but with a more reduced growth constant than for the noise free dynamics.

Transient spatiotemporal chaos exists in other reaction-diffusion systems or diffusively coupled map lattices that all have a stable state in the homogeneous system [3–5,9]. Similarities in these models motivate a comparative study of the effect of noise on the collapse of spatiotemporal chaos. The Baer-Eiswirth model [36], a realistic surface reaction model for the oxidation of CO on Pt, has common steady state features with the Gray-Scott model in the parameter regime of spatiotemporal chaos. The origin of spatiotemporal chaos, however, was reported to be different in these two models; a backfiring instability was identified in the Baer-Eiswirth model [36], and a Šilnikov-like orbit was identified in the Gray-Scott model [29]. Preliminary results suggest that spatially homogeneous noise can also advance the collapse of spatiotemporal chaos in the Baer-Eiswirth model. We expect that this is due to the existence of an unstable focus in both systems, since earlier studies have shown that dynamical noise decreases the average residence time of a trajectory in the neighborhood of an unstable steady state [34].

Transient spatiotemporal chaos is also discussed as a source for species extinction in theoretical ecologies [10,11]. An increasing interest in spatiotemporal ecological dynamics [37] stems from the identification of spatial symmetry [38] as a possible cause of extinction, although the origin of spatial synchronization is still under debate [39]. The Gray-Scott excitable dynamics in Eq. (1) phenomenologically mimics an ecological system, as it captures major ecological mechanisms like density-dependent species reproduction, competition for resource, a natural exponential species decay, and diffusion-based dispersal of species and resource. In realistic situations the ubiquitous presence of noise introduces further difficulties to determine the asymptotic stability of complex spatiotemporal ecologies. In this model dynamical noise can clearly delay, but also clearly advance the extinction of species depending on the amplitude and the degree of spatial inhomogeneity of the noise.

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[1] C. Grebogi, E. Ott, and J. A. Yorke, Physica D **7**, 181 (1983); H. Kantz and P. Grassberger, *ibid.* **17**, 75 (1985); T. Tel, in *Directions in Chaos III*, edited by H. Bai-lin (World Scientific, Singapore, 1990), pp. 149–211.
[2] G. Huber, P. Alstrom, and T. Bohr, Phys. Rev. Lett. **69**, 2380 (1992).
[3] A. Wacker, S. Bose, and E. Schöll, Europhys. Lett. **31**, 257 (1995).
[4] M. C. Strain and H. S. Greenside, Phys. Rev. Lett. **80**, 2306 (1998).
[5] R. Wackerbauer and K. Showalter, Phys. Rev. Lett. **91**, 174103 (2003).
[6] F. H. Willeboordse, Phys. Rev. E **47**, 1419 (1993).
[7] Y.-Ch. Lai and R. L. Winslow, Phys. Rev. Lett. **74**, 5208 (1995).
[8] J. P. Crutchfield and K. Kaneko, Phys. Rev. Lett. **60**, 2715 (1988).
[9] A. Politi, R. Livi, G. L. Oppo, and R. Kapral, Europhys. Lett. **22**, 571 (1993).
[10] K. McCann and P. Yodzis, Am. Nat. **144**, 873 (1994).
[11] L. Shulenburger, Y.-Ch. Lai, T. Yalcinkaya, and R. D. Holt, Phys. Lett. A **260**, 156 (1999).
[12] T. Hastings, Science **263**, 1133 (1994).
[13] K. Kaneko, Phys. Lett. A **149**, 105 (1990).
[14] S. Yonker and R. Wackerbauer, Phys. Rev. E **73**, 026218 (2006).
[15] R. Wackerbauer, H. Sun, and K. Showalter, Phys. Rev. Lett. **84**, 5018 (2000).
[16] J. Garcia-Ojalvo and J. M. Sancho, *Noise in Spatially Extended Systems* (Springer, New York, 1999).
[17] M. Loecher, *Noise-Sustained Patterns* (World Scientific, New Jersey, 2002).
[18] S. Kadar, J. Wang, and K. Showalter, Nature (London) **391**, 770 (1998).
[19] C. Zhou, J. Kurths, Z. Neufeld, and I. Z. Kiss, Phys. Rev. Lett. **91**, 150601 (2003).
[20] M. N. Lorenzo and V. Perez-Munuzuri, Chaos **11**, 371 (2001).
[21] F. H. Willeboordse and K. Kaneko, Phys. Rev. E **72**, 026207 (2005).
[22] H. Wang and Qi Ouyang, Phys. Rev. E **65**, 046206 (2002).
[23] H. Wang and Qi Ouyang, Chaos **15**, 023702 (2005).
[24] Y-Ch. Lai, Phys. Lett. A **200**, 418 (1995).
[25] F. Gassmann, Phys. Rev. E **55**, 2215 (1997); V. Paar and N. Pavin, *ibid.* **55**, 4112 (1997).
[26] J. A. Blackburn, N. Gronbech-Jensen, and H. J. T. Smith, Phys. Rev. Lett. **74**, 908 (1995).
[27] M. Franaszek, Phys. Rev. A **44**, 4065 (1991).
[28] P. Gray and S. K. Scott, Chem. Eng. Sci. **39**, 1087 (1984).
[29] J. H. Merkin, V. Petrov, S. K. Scott, and K. Showalter, Phys. Rev. Lett. **76**, 546 (1996); J. H. Merkin and M. A. Sadiq, IMA J. Appl. Math. **57**, 273 (1996).
[30] Y. Nishiura and D. Ueyama, Physica D **150**, 137 (2001).
[31] This diffusion constant $D=16$ in the ring network of coupled Gray-Scott excitable elements corresponds to $D=1$ in the discretized continuous space reaction-diffusion equation [5].
[32] For $N=120$ and $Q=0$ an increase of the ensemble size from 100 to 2000 different randomly chosen initial conditions changed the mean transient time by less than 1% [5]. For $N=120$ and $Q=0.001$ an increase from 40 random initial conditions to 100 random initial conditions changed the average lifetime by less than 3%.
[33] The average transient lifetime decreases with smaller t_0 for dichotomous noise, since smaller t_0 cause more perturbations of a trajectory per residence time. This increases the chance for a noise pattern with dominantly positive or negative noise events to initiate the collapse process (Fig. 6).
[34] R. Wackerbauer, Phys. Rev. E **59**, 2872 (1999); **58**, 3036 (1998).
[35] Spatially inhomogeneous noise with $r=2$ can also increase the average lifetime. Control calculations show that the average lifetime can be further increased by approximately another factor of 2, if the additional constraint applies than dichotomous perturbations with $r=2$ are antiphase in neighboring sets of network nodes. Above a critical noise amplitude, however, this additional constraint does not influence $\langle T \rangle$, which is again a consequence of large enough noise amplitudes favoring the extinction independent on the sign of the perturbation and the degree of spatial inhomogeneity (see Fig. 7).
[36] M. Bar and M. Eiswirth, Phys. Rev. E **48**, R1635 (1993).
[37] J. Bascompte and R. V. Sole, Trends Ecol. Evol. **10**, 361 (1995).
[38] M. Heino, V. Kaitala, and J. Lindstrom, Proc. R. Soc. London, Ser. B **264**, 481 (1997); D. J. D. Earn, P. Rohani, and B. T. Grenfell, *ibid.* **265**, 7 (1998).
[39] A. L. Lloyd and R. M. May, Trends Ecol. Evol. **14**, 417 (1999); E. Ranta, V. Kaitala, and J. Lindstrom, Proc. R. Soc. London, Ser. B **266**, 1851 (1999); T. G. Benton, C. T. Lapsley, and A. P. Beckerman, Ecol. Lett. **4**, 1 (2001).